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A NEW APPROACHES TO SOLVING FUZZY LINEAR SYSTEM WITH INTERVAL VALUED TRIANGULAR FUZZY NUMBER

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Abstract

In this paper we discussed fuzzy linear system with interval valued triangular fuzzy numbers. A new method for solving interval valued fuzzy linear system is proposed based upon the row reduced echelon form for solving the interval valued linear system of equations. The proposed method very easy to understand and apply for solving interval valued fuzzy linear systems occurring in real life situation and Numerical examples are given to illustrate the proposed method.

Key words: Fuzzy linear system, Triangular valued fuzzy number, etc.

Introduction:

One field of applied mathematics that has many applications in various areas of science is solving a system of linear equations. System of equations is the simplest and the most useful mathematical model for a lot of problems considered by applied mathematics. In practice, the exact values of coefficients of these systems are not a known role. This uncertainly may have either probabilistic or non probabilistic nature. Accordingly different approaches to the problem and different mathematical tools are needed. Therefore we need to develop numerical methods to find the roots of these systems.

Many results on interval valued fuzzy set have appeared. Basic theory for interval valued is discussed by [2]. Fully fuzzy linear system given by [1], fuzzy linear programming problems with interval valued fuzzy coefficient is given by [3] and the definitions and operations of interval valued fuzzy number are put forwarded by[4]

The structure of this paper is as follows, Section 2 begins with some basic definition about fuzzy numbers and a brief overview of the interval valued triangular fuzzy numbers. Section 3 defines interval valued fuzzy linear system using interval valued triangular fuzzy

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numbers and row reduced echelon form for solving fuzzy system of linear equations. Numerical examples to illustrate the proposed method are given in Section 4. This paper ends with a short conclusion in section 5.

Preliminaries:
In this section, we give some important definitions of fuzzy numbers are reviewed.

2.1 Fuzzy number
Fuzzy numbers are of great importance in fuzzy systems. The fuzzy numbers that usually used in applications are the triangular (shaped) and the trapezoidal (shaped) fuzzy numbers [8].

Definition 1: A fuzzy number is set like $u: R \to I = [0,1]$ which satisfies [8-12],

1. u is upper semi continuous,
2. $u(x) = 0$ outside some interval [c, d],
3. There are real numbers a, b such that $c \le a \le b \le d$ and

3.1 $u(x)$ is monotonic increasing on $[c,\ a]$
3.2 $u(x)$ is monotonic decreasing on $[b, d]$
3.3 $u(x) = 1,\ a \le x \le b$

The set of all these fuzzy numbers is denoted by $F(\Re)$E. An equivalent parametric is also given in [13].An alternative definition or parametric form of a fuzzy number which yields the same $F(\Re)$ is given by kelva[16].
Arithmetic operations between two triangular fuzzy numbers defined on universal set of real numbers $\Re$ are reviewed [18].

Definition 2:

A fuzzy number u in parametric form is a pair $(\underline{u}, \overline{u})$ of function $\underline{u}(r),\ \overline{u}(r)\ 0 \le r \le 1$, which satisfies the following requirements:

1. $\underline{u}(r)$ is a bounded monotonic increasing left continuous function
2. $\overline{u}(r)$ is a bounded monotonic decreasing left continuous function
3. $\underline{u}(r),\ \overline{u}(r)\ 0 \le r \le 1$

2.2 Fuzzy set
A fuzzy set on must possess at least the following three properties to qualify as a fuzzy number,
i $\tilde{A}$ must be a normal fuzzy set
ii ${}^{\alpha}\tilde{A}$ must be closed interval for every $\alpha \in [0 \quad 1]$
iii the support of $\tilde{A}$, ${}^{0+}\tilde{A}$ must be bounded.

2.3 Triangular Fuzzy Number: [18]

It is a fuzzy number represented with three points as follows: $\tilde{A} = (a_1, a_2, a_3)$
This representation is interpreted as membership functions and holds the following conditions

(i) a_1 *to* a_2 is increasing function

(ii) a_2 *to* a_3 is decreasing function

(iii) $a_1 \le a_2 \le a_3$

$$\mu_{\tilde{A}}(x) = \begin{cases} 0, & for\ x < a_1 \\ \dfrac{x - a_1}{a_2 - a_1}, & for\ a_1 \le x \le a_2 \\ \dfrac{a_3 - x}{a_3 - a_1}, & for\ a_2 \le x \le a_3 \\ 0, & for\ x > a_3 \end{cases}$$

2.4 Positive triangular fuzzy number: A positive triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = (a_1, a_2, a_3)$ where all a_i 's > 0 for all i=1, 2, 3.

2.5 Negative triangular fuzzy number: A negative triangular fuzzy number $\tilde{A}$ is denoted as $\tilde{A} = (a_1, a_2, a_3)$ where all a_i 's < 0 for all i=1, 2, 3.

Note: A negative Triangular fuzzy number can be written as the negative multiplication of a positive Triangular fuzzy number.

2.6 Equal Triangular fuzzy number: Let $\tilde{A} = (a_1, a_2, a_3)$ and $\tilde{B} = (b_1, b_2, b_3)$ be two triangular fuzzy numbers. If $\tilde{A}$ is identically equal to $\tilde{B}$ only if $a_1 = b_1,\ a_2 = b_2$ and $a_3 = b_3$.

2.7 Fuzzy Matrix:

A matrix $\tilde{A} = [\tilde{a}_{ij}]_{i,j=1}^{n}$ is called a fuzzy matrix if for all $i\ and\ j,\ \ \tilde{a}_{ij} \in F(\Re)$

$\tilde{A}$ Will be positive and denoted by $\tilde{A} > 0\ i\ and\ j,\ \ \tilde{a}_{ij} > 0$

$\tilde{A}$ Will be negative and denoted by $\tilde{A} > 0\ i\ and\ j,\ \ \tilde{a}_{ij} < 0$

2.8 Fuzzy Vector: A vector $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, ..., \tilde{x}_n)$ and denoted by $\tilde{X} \in F(\Re)$ is called fuzzy vector, where $\tilde{x}_i \in F(\Re),\ i = 1,2,3,...,n$

2.9 Fuzzy linear system:

The $n \times n$ linear system

$$(\tilde{a}_{11}\tilde{x}_1)+(\tilde{a}_{12}\tilde{x}_2)+(\tilde{a}_{13}\tilde{x}_3)+. \quad . \qquad .+(\tilde{a}_{1n}\tilde{x}_n)=\tilde{b}_1$$
$$(\tilde{a}_{21}\tilde{x}_1)+(\tilde{a}_{22}\tilde{x}_2)+(\tilde{a}_{23}\tilde{x}_3)+. \quad . \qquad .+(\tilde{a}_{2n}\tilde{x}_n)=\tilde{b}_{21}$$
$$(\tilde{a}_{31}\tilde{x}_1)+(\tilde{a}_{32}\tilde{x}_2)+(\tilde{a}_{33}\tilde{x}_3)+. \quad . \qquad .+(\tilde{a}_{3n}\tilde{x}_n)=\tilde{b}_3$$
$$.$$
$$.$$
$$(\tilde{a}_{n1}\tilde{x}_1)+(\tilde{a}_{n2}\tilde{x}_2)+(\tilde{a}_{n3}\tilde{x}_3)+. \quad . \qquad .+(\tilde{a}_{nn}\tilde{x}_n)=\tilde{b}_1$$

or in matrix form $\tilde{A}\tilde{x}=\tilde{b}$ is called a fuzzy linear system of equations

Where the coefficient matrix $\tilde{A}=\left[\tilde{a}_{ij}\right]_{i,j=1}^{n}$ is a fuzzy matrix and $\tilde{b}=\left(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3,......,\tilde{b}_n\right)^T$ is a fuzzy number and the fuzzy vector $\tilde{X}=(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,.....,\tilde{x}_n)$ is the unknown to be found.

2.2 .1 Interval valued fuzzy number: A interval valued fuzzy number $\tilde{u}$ is a pair of $[\underline{u},\overline{u}]$ of $\underline{u}(r),\ \overline{u}(r);\ 0\le r\le 1$ which satisfy the following conditions

Let $\tilde{V}=[\underline{v}(r),\overline{v}(r)]$

$x>0;\ \ x=[x\underline{v}(r),x\overline{v}(r)]$ and $x<0;\ \ x=[x\overline{v}(r),x\underline{v}(r)]$

$\tilde{V}+\tilde{U}=[\underline{v}(r)+\underline{u}(r),\ \overline{v}(r)+\overline{u}(r)]$

$\tilde{V}-\tilde{U}=[\underline{v}(r)-\overline{u}(r),\ \overline{v}(r)-\underline{u}(r)]$

2.2.2 Interval valued Triangular Fuzzy number:

If $\tilde{A}=[A^-,A^+],\ \tilde{B}=[B^-,B^+]$ are two interval valued triangular fuzzy number and $\tilde{A}^-=(a_1^-,a_2^-,a_3^-)$, $\tilde{A}^+=(a_1^+,a_2^+,a_3^+)$, $\tilde{B}^-=(b_1^-,b_2^-,b_3^-)$ and $\tilde{B}^+=(b_1^+,b_2^+,b_3^+)$ then for $\forall k\in R^+$

$\tilde{A}+\tilde{B}=[A^-+B^-,\ A^++B^+]$

$\tilde{A}-\tilde{B}=[A^--B^-,\ A^+-B^+]$

$k\tilde{A}=[kA^-,\ kA^+]$

Where $A^-\pm B^-=(a_1^-\pm b_1^-,\ a_2^-\pm b_2^-,a_3^-\pm b_3^-)$

$A^+\pm B^+=(a_1^+\pm b_1^+,\ a_2^+\pm b_2^+,a_3^+\pm b_3^+)$

$kA^-=(ka_1^-,\ ka_2^-,ka_3^-)$ and $kA^+=(ka_1^+,\ ka_2^+,ka_3^+)$

3. Proposed Method

One of the methods for solving the crisp linear system of equations $Ax=b$, is row reduced echelon form [20]. In this section the same method is extended to solve interval valued fuzzy linear system $\tilde{A}\tilde{x}=\tilde{b}$

Assuming $\tilde{A}=\left[\tilde{a}_{ij}\right]_{i,j=1}^{n}$ is a fuzzy matrix and $\tilde{b}=\left(\tilde{b}_1,\tilde{b}_2,\tilde{b}_3,......,\tilde{b}_n\right)^T$ is a fuzzy number and the fuzzy vector $\tilde{X}=(\tilde{x}_1,\tilde{x}_2,\tilde{x}_3,.....,\tilde{x}_n)$

We can write $\tilde{A}\tilde{x} = \tilde{b}$

$[\tilde{A}^-, A^+]\tilde{x} = [\tilde{b}^-, \tilde{b}^+]$

$$\left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{11}}\tilde{x}_1\right) + \left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{12}}\tilde{x}_2\right) + . \quad . + \left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{1n}}\tilde{x}_n\right) = \left(\underbrace{([b_1^-,b_1^+],[b_2^-,b_2^+],[b_3^-,b_3^+])}_{b_1}\right)$$

$$\left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{21}}\tilde{x}_1\right) + \left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{22}}\tilde{x}_2\right) + . \quad . + \left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{2n}}\tilde{x}_n\right) = \left(\underbrace{([b_1^-,b_1^+],[b_2^-,b_2^+],[b_3^-,b_3^+])}_{b_2}\right)$$

$$\left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{31}}\tilde{x}_1\right) + \left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{32}}\tilde{x}_2\right) + . \quad . + \left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{3n}}\tilde{x}_n\right) = \left(\underbrace{([b_1^-,b_1^+],[b_2^-,b_2^+],[b_3^-,b_3^+])}_{b_3}\right)$$

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$$\left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{n1}}\tilde{x}_1\right) + \left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{n2}}\tilde{x}_2\right) + . \quad . + \left(\underbrace{([a_1^-,a_1^+],[a_2^-,a_2^+],[a_3^-,a_3^+])}_{a_{nn}}\tilde{x}_n\right) = \left(\underbrace{([b_1^-,b_1^+],[b_2^-,b_2^+],[b_3^-,b_3^+])}_{b_n}\right)$$

or $[a_1]_{ij}\tilde{x} = [b_1]_i \Rightarrow A_1\tilde{x} = B_1$

$[a_2]_{ij}\tilde{x} = [b_2]_i \Rightarrow A_2\tilde{x} = B_2$

$[a_3]_{ij}\tilde{x} = [b_3]_i \Rightarrow A_3\tilde{x} = B_3$

Where $i, j = 1,2,,3,..,n$ and $[a_1] = [a_1^-, a_1^+], [a_2] = [a_2^-, a_2^+], [a_3] = [a_3^-, a_3^+]$

$[b_1] = [b_1^-, b_1^+], \; [b_2] = [b_2^-, b_2^+], \quad [b_3] = [b_3^-, b_3^+]$

Now to find the solution above system of equations

Compute the row reduced echelon form of the interval valued triangular augmented matrices (A_1, B_1), (A_2, B_2) *and* (A_3, B_3) by applying suitable row operations.

There may be following cases:

Case 1: If $rank(A_1) \neq rank(A_1, B_1)$ or $rank(A_2) \neq rank(A_2, B_2)$ or $rank(A_3) \neq rank(A_3, B_3)$ then the interval valued triangular fuzzy linear system of equations are inconsistent and no non negative solution exists.

Case 2: If $rank(A_1) = rank(A_1, B_1)$, $rank(A_2) = rank(A_2, B_2)$ and $rank(A_3) = rank(A_3, B_3)$ but there exists at least one negative element in the $(n+1)^{th}$ column of the row reduced echelon form of augmented matrices (A_1, B_1) $or (A_2, B_2)$ or (A_3, B_3), then the interval valued triangular fuzzy linear system of equations are inconsistent and no non negative solution exists.

Case 3: If $rank(A_1) = rank(A_1, B_1)$, $rank(A_2) = rank(A_2, B_2)$ and $rank(A_3) = rank(A_3, B_3)$ and all elements of $(n+1)^{th}$ column of the row reduced echelon form of augmented matrices (A_1, B_1) $or (A_2, B_2)$ or (A_3, B_3) are non negative, then the interval valued triangular fuzzy linear system of equations are consistent (i.e.). Non negative solution exists.

There may be following two sub cases

Case 3a: If $rank(A_1) = rank(A_1, B_1) \neq n$, $rank(A_2) = rank(A_2, B_2) = n$ and $rank(A_3) = rank(A_3, B_3) \neq n$ then the interval valued triangular fuzzy linear system of equations has infinite number of non negative solutions.

Case 3b: If $rank(A_1) = rank(A_1, B_1) = n$, $rank(A_2) = rank(A_2, B_2) \neq n$ and $rank(A_3) = rank(A_3, B_3) = n$ then the interval valued triangular fuzzy linear system of equations has unique and non negative solutions.

Compute the value of $\tilde{x}_i$, $i = 1,2,..,n$ using row reduced echelon form of augmented matrices (A_1, B_1) $or(A_2, B_2)$ or (A_3, B_3) respectively. The solution of the interval valued triangular fuzzy linear system of equations will be represented by $\tilde{X} = (\tilde{x}_1, \tilde{x}_2, \tilde{x}_3, ..., \tilde{x}_n)$

4. **Numerical Example:**

Consider the following interval valued triangular fuzzy linear system of equations and by proposed method.

$$(-2,3,4)(x_{1,}, y_1, z_1) + (-3,-2,-1)(x_{2,}, y_2, z_2) = (5,21,43)$$

$$(-1,1,2)(x_{1,}, y_1, z_1) + (1,3,4)(x_{2,}, y_2, z_2) = (-6,14,34)$$

$$\begin{bmatrix} (-2,3,4) & (-3,-2,-1) \\ (-1,1,2) & (1,3,4) \end{bmatrix} \begin{bmatrix} (x_{1,}, y_1, z_1) \\ (x_{2,}, y_2, z_{21}) \end{bmatrix} = \begin{bmatrix} (5,21,43) \\ (-6,14,34) \end{bmatrix}$$

We choose $\alpha \in (0,1)$

The interval valued triangular fuzzy linear system can be written as

$$\begin{bmatrix} ([-2-\alpha,-2+\alpha],[3-\alpha,3+\alpha],[4-\alpha,4+\alpha]) & ([-3-\alpha-3+\alpha],[-2-\alpha,2+\alpha],[-1-\alpha,-1+\alpha]) \\ ([-1-\alpha,-1+\alpha],[1-\alpha,1+\alpha],[2-\alpha,2+\alpha]) & ([1-\alpha,1+\alpha],[3-\alpha,3+\alpha],[4-\alpha,4+\alpha]) \end{bmatrix} \begin{bmatrix} (x_{1,}, y_1, z_1) \\ (x_{2,}, y_2, z_{21}) \end{bmatrix} = \begin{bmatrix} ([5-\alpha,5+\alpha],[21-\alpha,21+\alpha],[43-\alpha,43+\alpha]) \\ ([-6-\alpha,-6+\alpha],[14-\alpha,14+\alpha],[34-\alpha,34+\alpha]) \end{bmatrix}$$

Put $\alpha = 0.5$

$$\begin{bmatrix} ([-2.5,-1.5],[2.5,3.5],[3.5,4.5]) & ([-3.5,-2..5],[-2.5,-1.5],[-1.5,-0.5]) \\ ([-1.5,-0.5],[0.5,1.5],[1.5,2.5]) & ([0.5,1.5],[2.5,3.5],[3.5,4.5]) \end{bmatrix} \begin{bmatrix} (x_{1,}, y_1, z_1) \\ (x_{2,}, y_2, z_{21}) \end{bmatrix} = \begin{bmatrix} ([4.5,5.5],[20.5,21.5],[42.5,43.5]) \\ ([-6.5,-5.5],[13.5,14.5],[33.5,34.5]) \end{bmatrix}$$

The augmented Matrix

$$(A_1, b_1) = \begin{bmatrix} [-2.5,-1.5] & [-3.5,-2..5] & [4.5,5.5] \\ [-1.5,-0.5] & [0.5,1.5] & [-6.5,-5.5] \end{bmatrix}$$

$$(A_2, b_2) = \begin{bmatrix} [2.5,3.5] & [-2.5,-1.5] & [20.5,21.5] \\ [0.5,1.5] & [2.5,3.5] & [13.5,14.5] \end{bmatrix}$$

$$(A_3, b_3) = \begin{bmatrix} [3.5,4.5] & [-1.5,-0.5] & [42.5,43.5] \\ [1.5,2.5] & [3.5,4.5] & [33.5,34.5] \end{bmatrix}$$

The row reduced echelon form of this matrix is obtained as follows

$$(A_1,b_1)=\begin{bmatrix}[-2.5,-1.5] & [-3.5,-2..5] & [4.5,5.5]\\ [-1.5,-0.5] & [0.5,1.5] & [-6.5,-5.5]\end{bmatrix}$$ applying row operations, we get the resultant matrix

$$\begin{bmatrix}x_1\\ x_2\end{bmatrix}=\begin{bmatrix}[1.52,\ 3.1]\\ [-3.5,-3.2]\end{bmatrix}$$

Similarly the row reduced echelon form of this augmented matrix

$$(A_2,b_2)=\begin{bmatrix}[2.5,3.5] & [-2.5,-1.5] & [20.5,21.5]\\ [0.5,1.5] & [2.5,3.5] & [13.5,14.5]\end{bmatrix}$$ which gives $\begin{bmatrix}y_1\\ y_2\end{bmatrix}=\begin{bmatrix}[6.6,11.3]\\ [1.2,\ 3.1]\end{bmatrix}$

Finally the row reduced echelon form of this augmented matrix

$$(A_3,b_3)=\begin{bmatrix}[3.5,4.5] & [-1.5,-0.5] & [42.5,43.5]\\ [1.5,2.5] & [3.5,4.5] & [33.5,34.5]\end{bmatrix}$$ which gives $\begin{bmatrix}z_1\\ z_2\end{bmatrix}=\begin{bmatrix}[10.95,13.6]\\ [3.7,\ 12.3]\end{bmatrix}$

If $rank(A_1)=rank(A_1,B_1)=2$, $rank(A_2)=rank(A_2,B_2)=2$ and $rank(A_3)=rank(A_3,B_3)=2$ and all elements of $(n+1)^{th}$ column of the row reduced echelon form of augmented matrices (A_1,B_1) $or(A_2,B_2)$ or (A_3,B_3) are non negative, then the interval valued triangular fuzzy linear system of equations are consistent (i.e.). Non negative solution exists.

Conclusion:

In this paper, a new method is applied to find the solutions of interval valued fuzzy linear system of equations. Here triangular interval valued fuzzy number and row reduced echelon form of matrices is used to construct a new method for solving interval valued fuzzy linear system of equations and the validity of the proposed method is examined with numerical examples. The constructed method is efficient to determine the interval valued fuzzy linear system of equations occurring in the real life situations.

References:

1. Zadeh, L.A., The concept of a Linguistic variable and its applications to approximate reasoning – parts I, II and III", Inform. Sci. 8(1975) 199-249; 8 1975 301-357; 9(1976) 43-80.
2. R.L. Burden, J.D. Faires (2005). *Numerical Analysis*, eight editions. Thomson, USA.
3. George J. Klir, Ute St. Clair, Bo Yuan (1997). *Fuzzy Set Theory: Foundations and Applications*. Prentice Hall PTR.
4. J.J. Buckley, Y. Qu (1991). Solving fuzzy equations: a new solution concept. *Fuzzy Sets and System*. 39: 291-301.
5. S. Abbasbandy, B. Asady (2004). Newton's method for solving fuzzy nonlinear equations. *Appl Math Comput* 159: 349-356
6. S. Abbasbandy S, R. Ezzati (2006). Newton's method for solving a system of fuzzy nonlinear equations. *Appl Math Comput* 175: 1189-1199
7. S. Abbasbandy , A. Jafarian (2006). Steepest descent method for solving fuzzy nonlinear Equations. *Appl Math Comput* 175: 581-589

8. J.J. Buckley, E. Eslami and T. Feuring (2002). *Fuzzy Mathematics in Economics and Engineering*. Physica-Verlag
9. D. Dubois, H. Prade (1980). *Fuzzy sets and systems*. Theory and application. Academic, New York.
10. R. Goetschel, W. Voxman (1986). *Elementary Calculus*, Fuzzy Sets and Systems 18:31-43
11. J.E. Dennis, R.B Schnabel (1983). *Numerical Methods for Unconstrained Optimization and Nonlinear Equations*, Prentice-Hall Jersey.
12. J. Wright, Stephen, Nocedal, Jorge (2006). *Numerical Optimization*, 2e.pp (614)
13. H.J. Zimmermann (1991). *Fuzzy set theory and its application*. Kluwer, Dordrecht.
14. Kaufmann, A., Introduction to theory of Fuzzy Subsets, Vol. I (Academic Press, New York, 1975).
15. R. Goetschel, W. Voxman (1986). *Elementary Calculus*, Fuzzy Sets and Systems 18:31-43
16. Abbasbandy S., Ezzati R., Jafarian A.: LU decomposition method for solving fuzzy system of linear equations. Applied Mathematics and Computation **172**, 633–643 (2006)
17. Abbasbandy S., Jafarian A.: Steepest descent method for system of fuzzy linear equations, Applied Mathematics and Computation **175**, 823–833 (2006)
18. Abbasbandy S., Jafarian A., Ezzati R.: Conjugate gradient method for fuzzy symmetric positive-definite system of linear equations. Applied Mathematics and Computation **171**, 1184–1191 (2005)
19. Allahviranloo T.: Numerical methods for fuzzy system of linear equations. Applied Mathematics and Computation **155**, 493–502 (2004
20. Nasseri S.H., Sohrabi M., Ardil E.: Solving fully fuzzy linear systems by use of a certain decomposition of the coefficient matrix. International Journal of Computational and Mathematical Sciences, **2**, 140−142 (2008)
21. Nasseri S.H., Zahmatkesh F.: Huang method for solving fully fuzzy linear system of equations. The Journal of Mathematics and Computer Science, **1**, 1-5 (2010)
22. Lipschutz S.: Shaum's Outline of Theory and Problems of Linear Algebra, 3rd edition, McGraw Hill Book Company, New York, (2005).
23. Sun X., Guo S.: Solution to general fuzzy linear system and its necessary and sufficient condition. Fuzzy Information and Engineering **3**, 317-327 (2009)